(u, v), weight $w(u,v)$

$w(P) \quad \sum_{(u,v)\in P} w(u,v)$

$\delta(u,v) =$ u থেকে v এর মধ্যে শরটেস্ট পাথের লেংথ

18

$w(P) = \delta(u,v) = \infty$ যদি $u \xrightarrow{w(u,v)} v$

$u \not\to v$

$w(u,v) \in \mathbb{R}$

$w(u,v)$

Main Problem: given s (source), find for every other vertex $t \neq s$, $\delta(s,t)$

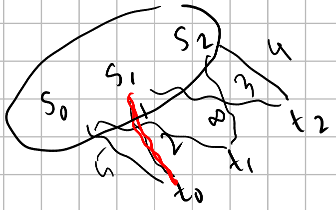
↳ weighted

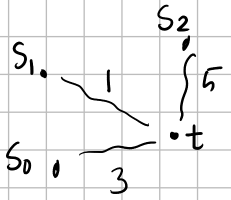
↳ unweighted

Variations:

(i) Multi-Source

(ii) Multi-Source → Multi destination

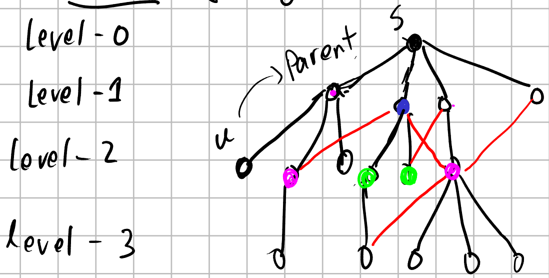




$O(V+E)$

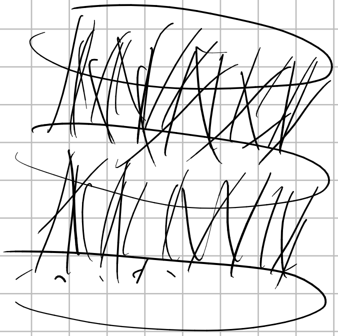
$O(E \log \underset{V}{E})$

BFS (Single Source)



Black = Tree edge
Red = Extra edge

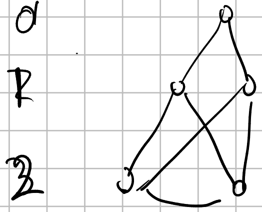
BFS tree

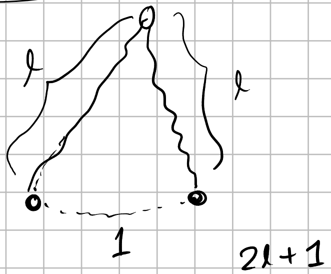


edge in same level ⇒ not bipartite

not bipartite ⇒ same level → edge

bipartite ⇒ no edge ∈ same level





BFS (Multisource)

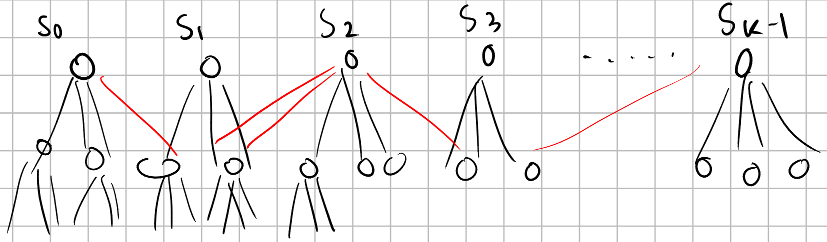
$s_0, s_1, s_2 \cdots s_{k-1}$

for each t, calculate $\underbrace{d[t]}_{} = \min_{x \in S} \left\{ \underbrace{\delta(x,t)}_{} \right\}$

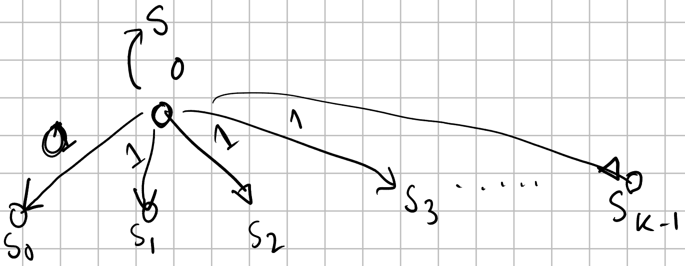
queue

$q \leftarrow s_0, s_1, s_2 \cdots s_{k-1}$, $d[s_0] = d[s_1] = d[s_2] = \cdots = d[s_{k-1}] = 0$

level



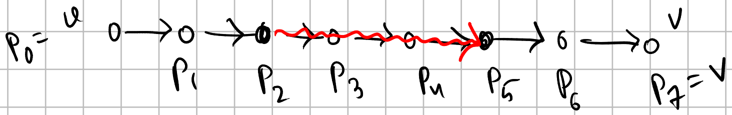
1 2 ⋯ n



Optimal Substructure of a Shortest Path

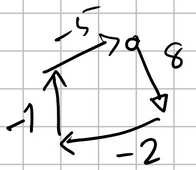
if p is a shortest path of u ⇝ v ,

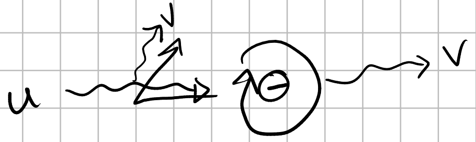
the red path is a shortest path of $P_2 \rightsquigarrow P_5$

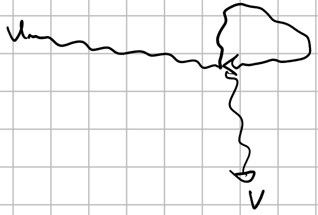


Negative weights

Negative Cycle : যেই সাইকেলের weight এর sum negative







$n = |V|$

$\#\text{edges} \leq |V| - 1$

$s.\pi = \text{NIL}$ parent

$d[s] = 0$

$V.\pi$

Relaxation u, v, w

$d[\cdots] = \infty$

$d[s] = 0$

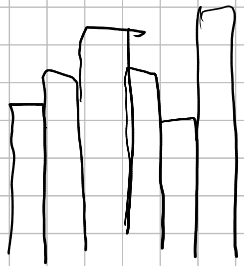
d $u \xrightarrow{w} v$

relax(u,v,w):

$d[v] = \min(d[v], d[u]+w)$

$d[v] \leq d[u] + w$

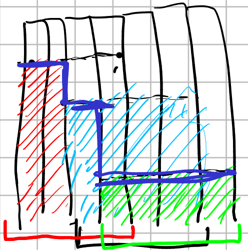
Arbitrarily choose an edge (u,v,w);
relax(u,v,w)



(l, r, x)

$a_i \leq x$ for each i

(l_i, r_i, x_i) →



```
for(i=l; i≤r; i++)
    $a_i = \min(a_i, x)$
```

$u \xrightarrow{w} v$

$d[u]$ = shortest Path length from s to u

2중 [illegible] $\left\{ d[v] > d[u] + w \right.$

condition $\left\{ d[v] \le d[u] + w \right.$ | each edge is not .vg but a condition

Single Source Shortest Path (Weighted Graph)

The following algorithm is correct:

Big Bc [① Iterate on the edges, and find an edge (u, v, w) such that $d[v] > d[u] + w$ and do, $d[v] = d[u] + w$

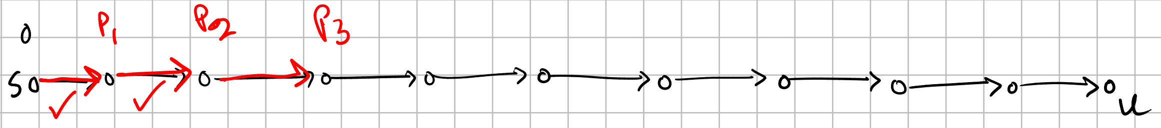
② Exit if you cannot find such an edge

→ T [illegible]

$O(\underset{|V|-1}{T} \times E)$ → $O(VE)$

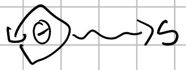
$\sum_u d[u]$

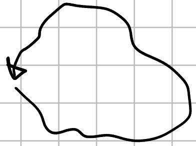
Spoiler: $T \le |V| - 1$



No negative cycle ⇒ you cannot find such an edge (u, v, w) that satisfies $\boxed{d[v] > d[u]+w}$ in the |V|-th application of "Big-R".

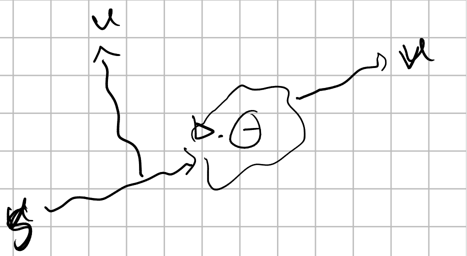
⇐





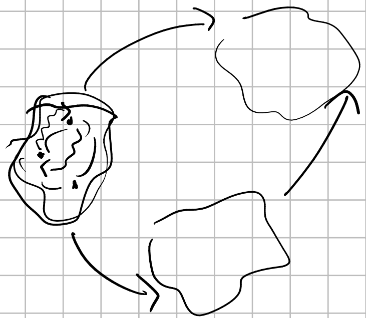
|P| ≥ V

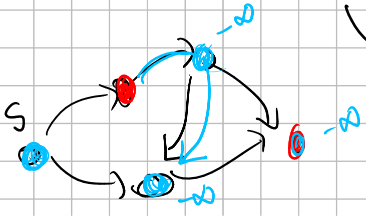
|V|

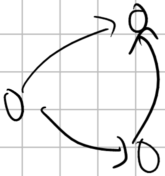


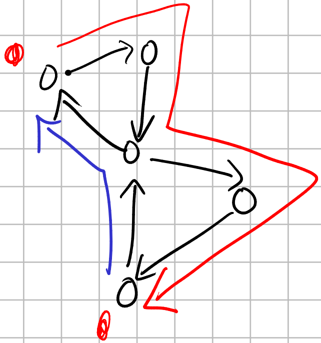
SCC

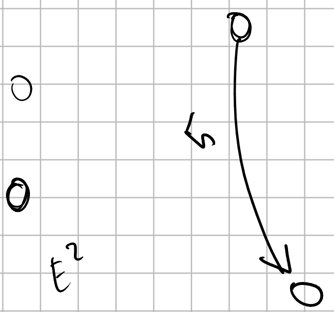
" Source [illegible] negative cycle reachable"

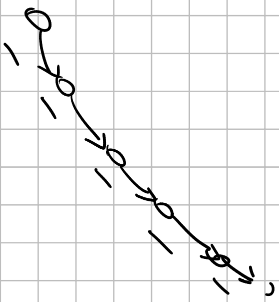










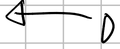


V

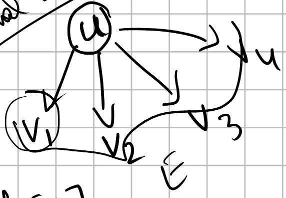
$d[v] > d[u] + w$

$d[v] = d[u] + w$

$\min\{d[u]\}$ ↳ u



$\boxed{\text{minimal } d[u]}$

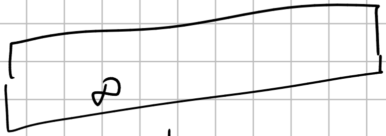


$T(V+E)$

↓

V

$(V+E)\lg V$



V

u

$d[v_1]$

$d[v_2]$

$d[v_3]$

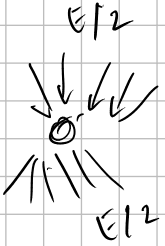
$d[v_1]$

$V^2 + E$

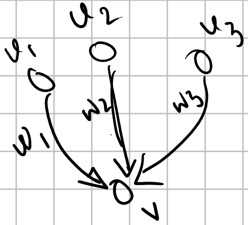
$\lg + |adj_u| \times \lg$

↳ $E \lg$

$V \lg$



$(E/2)^2 \lg$



$u_1 + w_1 = \delta(s, v)$

DAG